Electromagnetic Field Theory (EMT)

Lecture # 25

1) Transformer and Motional EMFs
2) Displacement Current
3) Electromagnetic Wave Propagation

Waves
&
Applications

Time Varying Fields

- Until now, we have restricted our discussions to static, or time invariant Electric and Magnetic fields

- Next, we shall examine situations where electric and magnetic fields are dynamic, or time varying

- It should be mentioned first that in static EM fields, electric and magnetic fields are independent of each other

- Whereas in dynamic EM fields, the two fields are interdependent

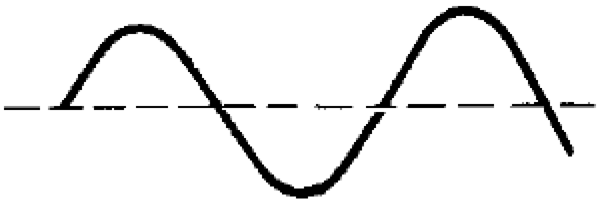
- In other words, a time-varying electric field necessarily involves a corresponding time-varying magnetic field

Time Varying Fields

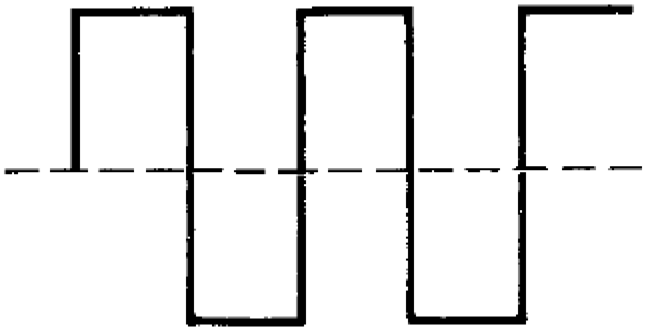
- Time-varying EM fields, represented by **E**(x, y, z, t) and **H**(x, y, z, t), are of more practical value than static EM fields
- Time-varying fields or waves are usually due to accelerated charges or time-varying currents such as sine or square waves
- Any pulsating current will produce radiation (time-varying fields)

Time Varying Fields

- Figure below shows examples of accelerated charges or time-varying currents



(a)



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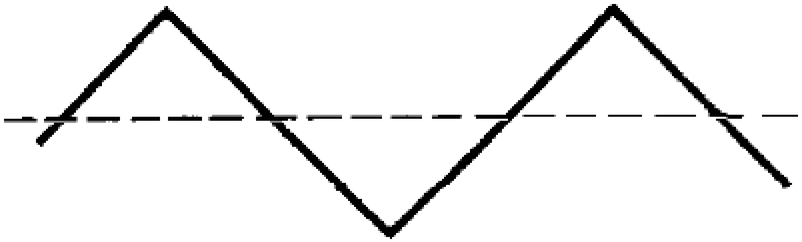

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Time Varying Fields

In summary:

- Stationary charges →Electrostatic fields
- Steady currents → Magneto-static fields
- Time-varying currents → electromagnetic fields (or waves)

Electromagnetic Field Theory (EMT)

Lecture # 24

1) Magnetic Torque and Momentum
2) Inductance and Magnetic Energy
3) Introduction to Time Varying Fields
4) Faraday's Law

Faraday’s Law

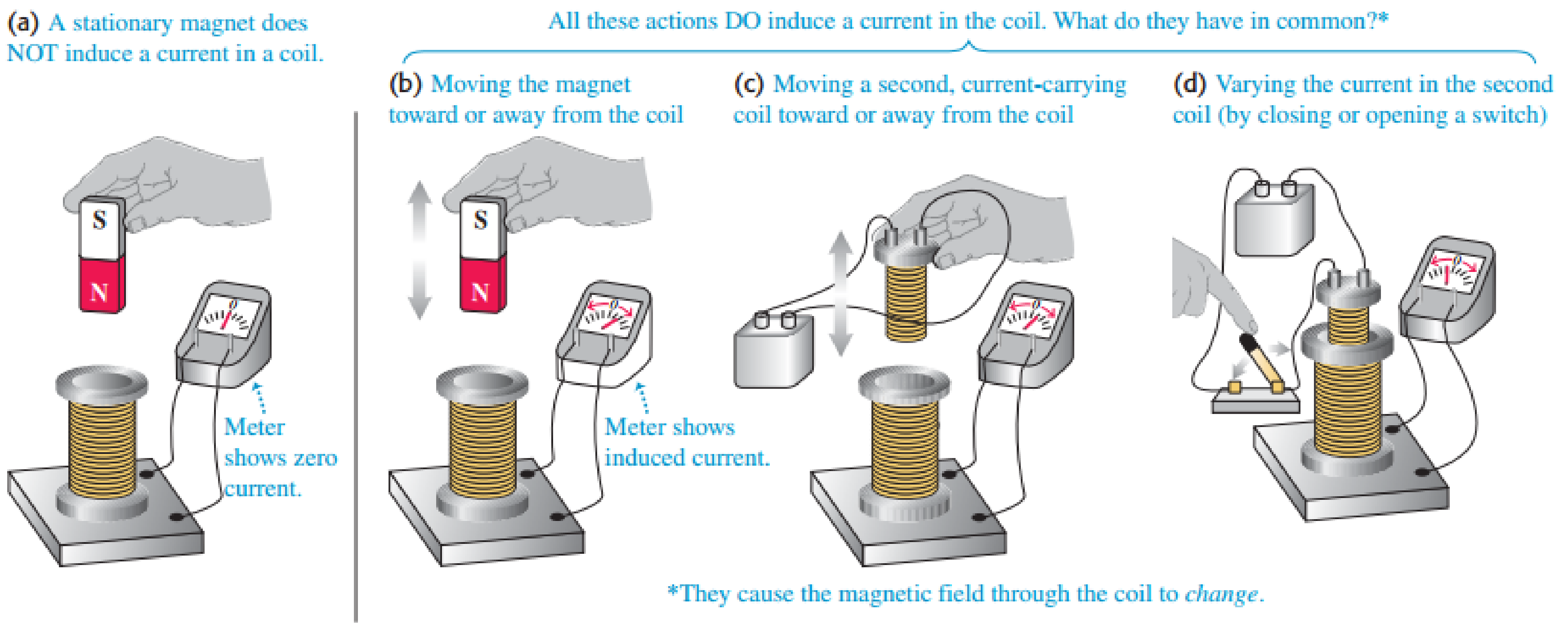
- After Oersted's experimental discovery (upon which Biot-Savart and Ampere based their laws) that a steady current produces a magnetic field, it seemed logical to find out if magnetism would produce electricity.

- In 1831, about 11 years after Oersted's discovery, Michael Faraday in London and Joseph Henry in New York discovered that a time-varying magnetic field would produce an electric current.

- According to Faraday's experiments, a static magnetic field produces no current flow, but a time-varying field produces an induced voltage (called electromotive force or simply emf) in a closed circuit, which causes a flow of current

Induction Experiments:

- But when we **move the magnet** either **toward or away** from the coil, the meter shows **current in the circuit** (Fig. b).
- If we keep the **magnet stationary** and **move the coil**, we **again detect a current** during the motion.
- We call this an induced current, and the corresponding emf required to cause this current is called an induced emf.



Faraday's Law

- The Faraday's law states that the induced emf, V_{emf} (in volts), in any closed **circuit** is equal to the time rate of change of the magnetic flux linkage by the circuit

- Mathematically, Faraday's law can be expressed as:

$$V_{\text{emf}} = -\frac{d\lambda}{dt} = -N\frac{d\Psi}{dt}$$

- where N is the number of turns in the circuit and Ψ is the flux through each turn

- *Lenz's law* states that the direction of current flow in the circuit is such that the induced magnetic field produced by the induced current will oppose the original magnetic field.

Faraday's Law

- From Lenz's law, the negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it

- Recall that we described an electric field as one in which electric charges experience force.

- The electric fields considered so far are caused by electric charges; in such fields, the flux lines begin and end on the charges

- There are other kinds of electric fields not directly caused by electric charges

- These are emf-produced fields

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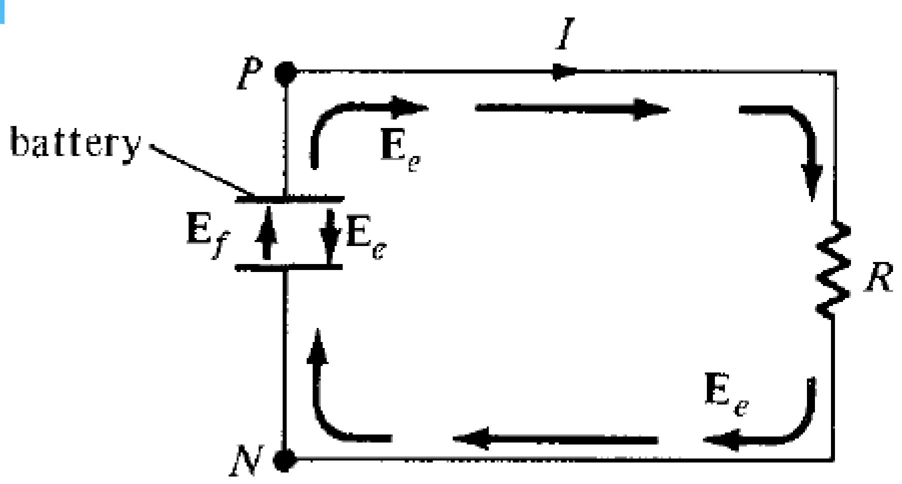
Electromotive Force (emf)

- Consider the electric circuit in figure below, where the battery is a source of emf



- The electrochemical action of the battery results in an emf-produced field $\mathbf{E}_f$

- Due to the accumulation of charge at the battery terminals, an electrostatic field $\mathbf{E}_e$ $(-\nabla V)$ also exists

Electromotive Force (emf)

- The total electric field at any point is:

$$\mathbf{E} = \mathbf{E}_f + \mathbf{E}_e$$

- Note that $\mathbf{E}_f$ is zero outside the battery
- $\mathbf{E}_f$ and $\mathbf{E}_e$ have opposite directions in the battery
- The direction of $\mathbf{E}_e$ inside the battery is opposite to that outside it
- By integrating the above equation over the closed circuit, we get:

$$\oint_L \mathbf{E} \cdot d\mathbf{l} = \oint_L \mathbf{E}_f \cdot d\mathbf{l} + 0 = \int_N^P \mathbf{E}_f \cdot d\mathbf{l} \qquad \text{(through battery)}$$

- Where $\oint \mathbf{E}_e \cdot d\mathbf{l} = 0$ because $\mathbf{E}_e$ is conservative

Electromotive Force (emf)

- The emf of the battery is the line integral of the emf-produced field, that is:

$$V_{\text{emf}} = \int_N^P \mathbf{E}_f \cdot d\mathbf{l} = -\int_N^P \mathbf{E}_e \cdot d\mathbf{l} = IR$$

- The negative sign is because $\mathbf{E}_f$ and $\mathbf{E}_e$ are equal but opposite within the battery

- It is important to note that:

- An electrostatic field $\mathbf{E}_e$ cannot maintain a steady current in a closed circuit since

$$\oint_L \mathbf{E}_e \cdot d\mathbf{l} = 0 = IR$$

- An emf-produced field $\mathbf{E}_f$ is non-conservative.

Transformer and Motional EMFs

- After considering the connection between emf and electric field, we now examine how Faraday's law links electric and magnetic fields
- For a circuit with a single turn (N = 1), we have:

$$V_{\text{emf}} = -\frac{d\Psi}{dt}$$

- In terms of **E** and **B**, the above equation may be written as:

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S}$$

- where Ψ has been replaced by $\int_S \mathbf{B} \cdot d\mathbf{S}$ and S is the surface area of the circuit bounded by the closed path L

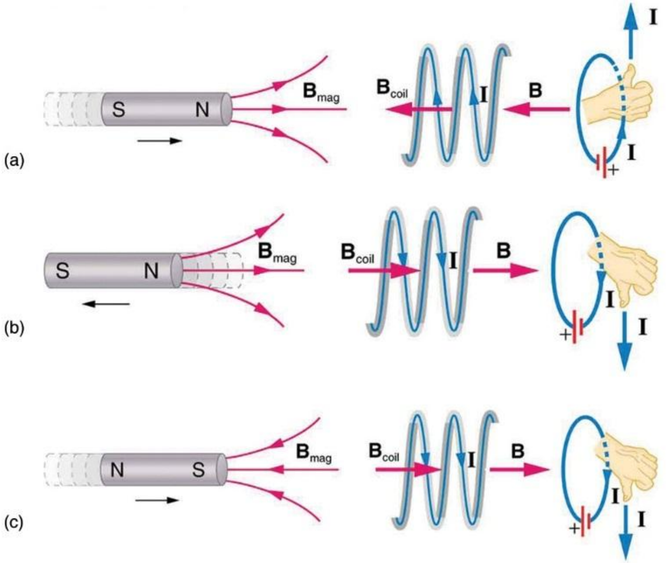
Transformer and Motional EMFs

- It is clear from above equation that in a time-varying situation, both electric and magnetic fields are present and are interrelated

- The variation of flux with time (as in previous equation) may be caused in three ways:

1. By having a stationary loop in a time-varying **B** field
2. By having a time-varying loop area in a static **B** field
3. By having a time-varying loop area in a time-varying **B** field

- Each of these will be considered separately.



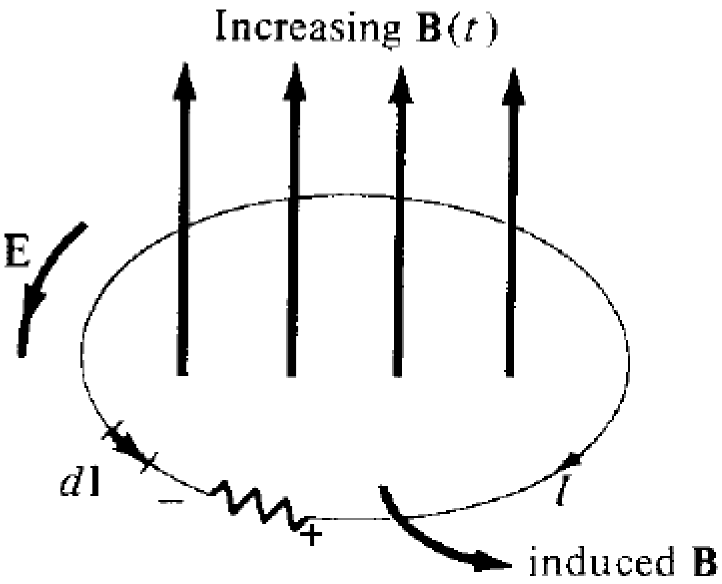
Stationary Loop; Time-Varying B Field

- Figure below shows a stationary conducting loop in a time varying magnetic **B** field
- The emf is given as:

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

- This emf induced by the time-varying **B** field in a stationary loop is often referred to as transformer emf in power analysis since it is due to transformer action

- Observe in the figure that the Lenz's law is obeyed; the induced current *I* flows such as to produce a magnetic field that opposes **B**(t)



Stationary Loop; Time-Varying B Field

- By applying Stokes theorem to the emf equation, we obtain:

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

- Therefore, we get:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- This is one of the Maxwell's equations for time-varying fields

- It shows that the time varying **E** field is not conservative

$$(\nabla \times \mathbf{E} \neq 0)$$

- This implies that the work done in taking a charge about a closed path in a time-varying electric field, for example, is due to the energy from the time-varying magnetic field

Moving Loop; Static B Field

- When a conducting loop is moving in a static **B** field, an emf is induced in the loop
- Recall that the force on a charge moving with uniform velocity **u** in a magnetic field **B** is:

$$\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$$

- We define the motional electric field $\mathbf{E}_m$ as:

$$\mathbf{E}_m = \frac{\mathbf{F}_m}{Q} = \mathbf{u} \times \mathbf{B}$$

- If we consider a conducting loop, moving with uniform velocity **u** as consisting of a large number of free electrons, the emf induced in the loop is:

$$V_{\text{emf}} = \oint_L \mathbf{E}_m \cdot d\mathbf{l} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

Moving Loop; Static B Field

- This type of emf is called motional emf or flux-cutting emf because it is due to motional action

- It is the kind of emf found in electrical machines such as motors, generators, and alternators.

Moving Loop; Time varying B Field

- In this case, both transformer emf and motional emf are present

- Hence we combine both the emfs as:

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

Maxwell equations after faraday's law

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \quad \text{(Gauss's law)},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0 \quad \text{(no name)},$$

$$\text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(Faraday's law)},$$

$$\text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \text{(Ampère's law)}.$$

Electromagnetic Field Theory (EMT)

Lecture # 25

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3) Electromagnetic Wave Propagation

Displacement Current

- Maxwell's curl equation for static EM fields is:

$$\nabla \times \mathbf{H} = \mathbf{J}$$

- But the divergence of the curl of any vector field is identically zero, hence:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J}$$

- The continuity of current equation, however, requires that:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \neq 0$$

- Thus the above equations are obviously incompatible for time-varying conditions

- We must modify Maxwell's curl equation to agree with the continuity equation

Displacement Current

- To do this, we add a term to Maxwell's curl equation so that it becomes:

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d$$

- where $\mathbf{J}_d$ is to be determined and defined

- Again, the divergence of the curl of any vector is zero, hence:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{J}_d$$

- In order for the above equation to agree with the continuity equation:

$$\nabla \cdot \mathbf{J}_d = -\nabla \cdot \mathbf{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}) = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t}$$

- Or:

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{dt}$$

Displacement Current

- Substituting $\mathbf{J}_d$ into Maxwell's curl equation, we get:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

- This is Maxwell's equation (based on Ampere's circuit law) for a time-varying field

- The term $\mathbf{J}_d$ is known as *displacement current density* and $\mathbf{J}$ is the conduction current density

- The insertion of $\mathbf{J}_d$ into Maxwell's curl equations was one of the major contributions of Maxwell

- Without the term $\mathbf{J}_d$, electromagnetic wave propagation (radio or TV waves, for example) would be impossible

Displacement Current

- At low frequencies, $\mathbf{J}_d$ is usually neglected compared with $\mathbf{J}$, however, at radio frequencies, the two terms are comparable

- At the time of Maxwell, high-frequency sources were not available and the curl equation could not be verified experimentally

- It was years later that Hertz succeeded in generating and detecting radio waves thereby verifying the curl equation

- This is one of the rare situations where mathematical argument paved the way for experimental investigation.

Displacement Current

- Based on the displacement current density, we define the *displacement current* as:

$$I_d = \int \mathbf{J}_d \cdot d\mathbf{S} = \int \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$$

- It must be kept in mind that displacement current is a result of time-varying electric field

- A typical example of such current is the current through a capacitor when an alternating voltage source is applied to its plates

Maxwell's Equations

- For a field to be "qualified" as an electromagnetic field, it must satisfy all four Maxwell's equations

Differential Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of isolated magnetic charge*
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = -\dfrac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{S}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$	Ampere's circuit law

*This is also referred to as Gauss's law for magnetic fields.

Problem-1

- In free space, **E** = 20 cos *(wt* **-** 50x) **ay** V/m. Calculate
- (a) **J***d*
- (b) **H**
- (c) *w*